Policy Components of Arms Competition

Mathew D. McCubbins, California Institute of Technology and University of Texas at Austin

This report suggests and justifies a simple approach to arms competitions, wherein arms competitions are viewed as disaggregated competitions between pairs of weapons systems for executing mutually incompatible policy goals. This approach is derived from a decision theoretic model of armament choice, in which military decision makers make trade-offs between alternative strategies of weapons deployment to achieve national foreign policy objectives. Data representing a cross section of the U.S. and USSR military arsenals are employed in a quasi first-difference two-stage least squares analysis to provide evidence for the propositions of the model and this approach.

Introduction

Since the pioneering work of Lewis Richardson, the study of arms races has formed a central part of the formal research in international relations. Much of this work, however, has been disappointing; the statistical results have been rather unimpressive. The research reported here is based on the premise that a major reason for these poor results is an inappropriate use of aggregation. Employing a decision theoretic framework—one that is not inconsistent with the rational actor, bureaucratic, and organizational paradigms of political science—this study developed a testable model of arms races that makes explicit the relation of arms growth to the achievement of a nation's foreign policy goals and, moreover, yields stronger statistical results than much of the previous work in this field. Implicit in this approach is an intriguing counterintuitive proposition: arms control itself can be a cause of arms races.

Previous Research

The model formulated by Richardson is essentially a descriptive model. Its primary contribution is that it encompasses much of our intuition about the causes and development of arms competitions. It posits arms races as competition between two mutually distrustful nations, competition in which military budget appropriations or military buildups by one nation are answered in kind by the competing nation or nations. According to this model, competitive increases continue indefinitely until abated by the limits of wealth of the competing nations or by war.

The basic Richardson model is well known (cf. Richardson, 1960; Rapoport 1957, 1960). Other scholars have discussed Richardson stability conditions and have extended the model to a multination case (O'Neil, 1970). Alternative models similar to Richardson's have been proposed, and the relation between arms races and war initiation have been investigated (Caspary, 1967; Friberg and Jonsson, 1968; Intriligator, 1964).
The most plausible operationalization of the Richardson model is as a difference equation (Equation 1). This formulation describes nations $X$'s and nations $Y$'s stock of weapons (or military budgets) at time $t$ ($X_t$ and $Y_t$, respectively) as a function of both their own previous stock of weapons ($X_{t-1}$ and $Y_{t-1}$, respectively) and their adversary's previous stock of weapons ($Y_{t-1}$ and $X_{t-1}$):

$$
X_t = \alpha X_{t-1} + \beta Y_{t-1} + c,
$$

$$
Y_t = \delta X_{t-1} + \gamma Y_{t-1} + f,
$$

(1)\[\beta, \delta > 0 \text{ and } \alpha, \gamma < 0\]

The primary problem with this model arises when trying to estimate it, because it is not clear what $X$ and $Y$ should stand for in the arms race context. Richardson thought of them as measures of the "total armed might" of the two mutually distrustful countries and later tested the model with yearly defense budgets as proxies for $X$ and $Y$. Most subsequent analyses have also employed defense budgets (see Chatterjee, 1974; Lambelet, 1976; Ruloff, 1975; Taagepera, Shiffler, Perkins, and Wagner, 1975). The model has further been applied to modern treatments of armament races in the missile age (see Burns, 1959; Boulding, 1961; Brito and Intriligator, 1977; Intriligator and Brito, 1976; Luterbacher, 1965, 1977; Saaty, 1968; and Taagepera, 1976). It should be noted that many of these later studies applied the Richardson model to stocks of weapons rather than to defense budgets.

The interpretation of defense budgets as a nation's "total armed might" is not entirely unreasonable, because increases in military budgets necessarily precede increases in "total armed might." However, as evidenced in many of the above-listed studies, such a proxy gives no indication of the putative arms competition between the two superpowers. Indeed, these previous investigations have found little support for the existence of Richardson arms races. Many scholars argued that the data employed (military budgets) was too clumsy to allow estimation of often small incremental changes in "total armed might." Indeed, aggregate measures such as budgets could well mask small incremental changes in weapons systems, and there is no reason to expect total military budgets between two (or more) competing nations to be linked in a Richardson (or any other) fashion.

But this analysis argues that the problem is more fundamental and arises from the use of aggregate data for the study of arms races. If many individual (disaggregated) arms races occur simultaneously between two countries, they may all be correlated, or they may "heat up" and "cool down" independently. In the first case, one would observe the classical sort of aggregate arms race typically considered in the prevailing literature. In the second case, races related to various policy conflicts
might cancel each other out, in which case aggregate arms stocks or military expenditures are constant, even though strenuous weapons competition is actually occurring.

As indicated earlier, other scholars have examined the Richardson process through an analysis of the total stocks of weapons possessed by each competing nation. Such an analysis, it was thought, might capture the subtle year-to-year changes in armaments that we expect to observe. However, these approaches were often merely a misapplication of disaggregated data, a misapplication brought about by the poor conceptualization inherent in the Richardson model. Such armament studies have frequently centered on competitions between complementary weapons as exemplified in Figure 1. Figure 1 depicts Soviet and American stocks of manned strategic bombers and registers the perceived decrease in American and Soviet bomber strength. However, as is true of many of the disaggregate studies it exemplifies, no arms competition is evident in Figure 1.

Stated most boldly, the basic aggregation problem inherent in the Richardson formulation derives from the fact that different weapons systems possess different policy characteristics. Each weapons system, whether it be a Marine Corps infantry battalion or a MX missile squadron, has a policy mission for which it was designed and produced to fulfill. To be sure, such systems are often multipurpose, but the recognition of such policy missions is central to understanding and defining arms competitions. The Richardson formulation, by not explicitly considering these policy characteristics of weapons systems, is unable to discriminate between which groups of weapons we should and should not expect to observe in competition. By employing a decision theoretic approach this study incorporates the policy characteristics of weapons into a resulting theory of arms competition.

**A Decision Theoretic Approach to Arms Competition**

In the previous section it was argued that the terms of the Richardson arms race equations have been poorly operationalized. In this section it will be suggested that, to the extent that nations engage in arms races, it does not seem likely that they operate at either aggregate levels (total defense expenditures) or at complementary disaggregations (e.g., bombers against bombers); to the extent that they race at all, it seems more likely that nations procure arms that are best suited for offsetting an adversary’s recent arms acquisitions (e.g., stepped-up bomber deployment by one nation will trigger new deployment of interceptors by the nation’s adversary). This, it will be shown, is the result of rational actions by

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2 Other problems related to the asymptotic properties of the coefficients of the Richardson model have been discussed elsewhere (Ferejohn, 1976; Schrodt, 1978).
FIGURE 1
Soviet and American Strategic Bombers
(Note Differing Scales)
cost-conscious decision makers. The mathematical formulation that follows established this result. The basis for the estimation is derived in the following section.

The "total armed might" of a nation is a direct extension of that nation's foreign policy objectives and its overall strategic doctrine. These foreign policy objectives dictate the size and shape of the military force a nation will develop. A nation's strategic doctrine identifies the types of responses, missions, and tasks its military force must be designed to fulfill. Each weapons system procured then fulfills a specific policy mission as necessitated by the needs related to the nation's strategic doctrine.

For example, one American foreign policy objective is the prevention of nuclear conflict. A strategic doctrine developed relative to this objective is mutual deterrence. Specific weapons developed to fulfill policy missions under this doctrine are land-based intercontinental ballistic missiles (ICBMs), manned strategic bombers, and sea-based submarine-launched ballistic missiles (SLBMs). Each has a policy mission (i.e., that of inflicting or threatening to inflict a nuclear strike on point targets).

Nations derive political gain from the use, or potential use, of their "total armed might" in accordance with their strategic doctrine. The basic behavioral postulate of the decision theoretic model to be put forth here is that military decision makers select weapons systems and procure armaments to maximize their capability to pursue their nation's foreign policy goals. Such choices, of course, are subject to their nation's doctrinal, production, budgetary, and technological constraints. Given this behavioral assumption, I will define a set of refutable hypotheses relating arms race behavior and arms control to the decision calculus just mentioned.

More formally, the behavioral assumption we posit is that a military decision maker engages in some sort of constrained maximizing behavior, the objective of which (for the two-nation case \( A \) and \( B \)) is to maximize

\[
\Pi^4 (q_1, \ldots, q_n, x_1, x_2, w_1, \ldots, w_n),
\]

where \( q_1, \ldots, q_n \) represents weapons allocations for country \( A \), given a set of specific foreign policy goals; \( x_1 \) and \( x_2 \) represent inputs to the production of the above weapons systems; \( w_1, \ldots, w_n \) represents the weapons allocations chosen by an adversary country \( B \), given its own set of foreign policy goals; and \( \Pi^4(\cdot) \) represents the decision makers' political gain or profit from deploying \( q_1, \ldots, q_n \). This maximization is subject

\(^3\) The political gain (or profit) a nation derives from the deployment of a specific weapons system can consist of a combination of foreign policy and domestic political gain. No assumption is made concerning the content of political gain, inasmuch as it is employed merely to represent the returns a nation receives from weapons deployment.
to the production and technology constraints inherent in nation A's economy, which we will summarize as the implicit production constraint

\[ F(q_1, \ldots, q_n, x_1, x_2) = 0. \] (3)

Thus I postulate each nation maximizes its own political gain \( \Pi(\cdot) \) by selecting, in an optimal fashion, the deployment levels for each weapons system in its choice set \( q_1, \ldots, q_n \) and the employment levels of productive inputs to armament manufacture \( x_1 \) and \( x_2 \), given their policy objectives and with respect to the choices of their adversary \( w_1, \ldots, w_n \). This maximization is performed with respect to \( q \) and \( x \), the armament levels deployed, and the production inputs employed, taking \( w \), the adversary's armament level, as a parameter.

The approach outlined here does not assume or depend on any formulation of government behavior. The model is consistent with, or at least not inconsistent with, the rational actor, bureaucratic, or organizational frameworks developed by Allison (1971). To be sure, the decision calculus is most readily appreciated as a two-person (i.e., two-nation) model and as such fulfills a rational actor framework of government. However, the interactions of various bureaucracies or the consequences of standard operating procedures can result in actions that taken altogether appear as if the bureaucracy or organization was acting to maximize political gain as asserted.

A necessary consequence of the behavior assumed above in Equations two and three is that the first-order partial derivatives of the following Lagrangian equal zero:

\[ L = \sum_j \Pi_j + \lambda F \]

where \( \lambda \) is the Lagrange multiplier. Employing a set of very general assumptions concerning government behavior and arms growth (see McCubbins, 1979) and applying well-known comparative statics techniques, we can derive several testable hypotheses concerning arms competition. These propositions follow directly from the maximization in Equation two, performed individually and independently by each nation.

The refutable hypothesis that concerns us here indicates between which groups of weapons we should and should not expect to observe competition for nations with conflicting foreign policy goals:

* In general, the bureaucratic and organizational approaches to modeling can be based upon models of rational individual behavior, wherein the actor is a bureaucrat or wherein the organizational structure of the decision-making unit influences or defines the choices of the individual actors.
Refutable Hypothesis: Arms competitions develop only between weapons systems that are endowed with conflicting policy missions by their nation's strategic doctrine and goals.

Formally, the refutable hypothesis states that the rate of change of the \(j^{th}\) weapons system for nation \(A\), which is \(q_j\), with respect to changes in the \(k^{th}\) weapons system for nation \(B\), which is \(w_k\), is positive if the weapon's policy tasks are incompatible options for each other \((j = k)\) and is zero if the weapons are noncompetitive options \((j \neq k)\) for each other (McCubbins, 1979, pp. 5-16):

\[
\frac{\partial q_j}{\partial w_k} > 0 \text{ if } j = k; \quad \frac{\partial q_j}{\partial w_k} = 0 \text{ if } j \neq k. \tag{4}
\]

The model thus makes explicit the relation of arms growth to achieving foreign policy goals by the military force decision makers. The hypothesis suggests that arms races should be viewed as competitions between pairs of weapons systems for achieving incompatible policy goals and not as competitions between the aggregate "armed might" of two mutually distrustful nations.

Estimation

The basic hypothesis to be examined here is that arms competitions, between two reciprocally antagonistic countries, occur only between weapons systems possessed of mutually incompatible policy goals. The corollary to be examined is that arms competitions will not occur between weapons systems possessed of congenial policy goals:

\(H_1: \text{Arms competitions occur between weapons systems with incompatible policy goals.}\)

\(H_2: \text{Arms competitions do not occur between weapons systems with harmonious policy goals.}\)

Hypotheses \(H_1\) and \(H_2\) represent the propositions derivable from the decision theoretic model as postulated in Equation four. American and Soviet stocks of weapons will be employed to test these hypotheses. The decision theoretic model, with its focus on the policy characteristics of weapons systems, can be formulated as an \(n\)-person general sum game, reducible to a Nash bargaining game for the two-nation case (McCubbins, 1979). The objective function in Equation two posits that both nations will simultaneously determine their optimal stocks of weapons, given their policy objectives and constraints and taking into account the simultaneous decisions and policy objectives of their adversary.
Estimation of hypotheses $H_1$ and $H_2$, then, necessitates estimating a simultaneous equation system for each pair of opposing weapons systems. The endogenous variables in the following analysis are, then, the actual stocks of various weapons systems deployed by the United States and the Soviet Union in their putative arms competition. The exogenous (independent) variables are the respective gross national products (GNPs) of each nation, where GNP is taken as a proxy for the production, budgetary, and technological constraints of each nation.\(^5\)

The sample of weapons systems, as listed in Table 1, contains a cross section of the conventional arsenals of the two superpowers, with the exception of manned strategic bombers (for reasons to be explained below). The sampling reflects the specificity with which the policy characteristics of weapons systems can be identified and the availability of quality time series on each weapons system. Weapons systems with more or less singular policy objectives that also had lengthy time series available for estimation were selected. On the whole, then, this excludes strategic missiles and naval warships because they serve a multitude of purposes. Having many purposes, by itself, however, does not present important theoretical problems, but rather presents problems with identifying the simultaneous system and with estimating a large number of right-hand side variables with a shortage of degrees of freedom.

Strategic interceptors, surface-to-air missiles (S.A.M.s) and, antitank missiles have very specific and well-defined policy missions. Tanks and tactical aircraft have less specifically identifiable policy missions, but it would seem reasonable to expect that their numbers grow in relation to the number of antitank and antiaircraft weapons deployed by their adversary and vice versa. Manned strategic bombers and strategic interceptors present a similar situation; given a policy objective to be fulfilled, the stock of strategic bombers should increase in relation to the deployed stock of strategic interceptors by the adversary.

Figure 2 displays the relationship between the deployed levels of Soviet heavy and medium tanks and American antitank missiles. The figure suggests that the rapid and exponential deployment of antitank

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GNP figures for the United States and the USSR were found in *World Armaments and Disarmaments* and the *Military Balance*. The primary functions of weapons systems were drawn largely from Collins and from N. Polmar, *Strategic Weapons: An Introduction* (New York: National Strategy Information Center, Inc., 1975).
POLICY COMPONENTS OF ARMS COMPETITION

TABLE 1

Weapons Systems

<table>
<thead>
<tr>
<th>Sample of Soviet and American Weapons Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
</tr>
<tr>
<td>U.S. strategic interceptors (1964–76)</td>
</tr>
<tr>
<td>U.S. tactical S.A.M. (1968–76)</td>
</tr>
<tr>
<td>U.S. tactical aircraft (1968–76)</td>
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<tr>
<td>U.S. heavy and medium tanks (1968–76)</td>
</tr>
<tr>
<td>U.S. antitank missiles (1966–76)</td>
</tr>
<tr>
<td>U.S. manned strategic bombers (1964–76)</td>
</tr>
<tr>
<td>USSR</td>
</tr>
<tr>
<td>Soviet strategic interceptors (1964–76)</td>
</tr>
<tr>
<td>Soviet tactical S.A.M. (1968–76)</td>
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<tr>
<td>Soviet tactical aircraft (1968–76)</td>
</tr>
<tr>
<td>Soviet heavy and medium tanks (1966–76)</td>
</tr>
<tr>
<td>Soviet antitank missiles (1968–76)</td>
</tr>
<tr>
<td>Soviet manned strategic bombers (1964–76)</td>
</tr>
</tbody>
</table>

Missiles by the United States is a response to the ever-increasing number of Soviet tanks deployed. Interestingly, Figures 3 and 4 map mirror images of a disarmament race by the superpowers in manned strategic bombers and strategic interceptors. Possibly instigated by the advent of long-range ICBMs and SLBMs, both superpowers, over the period of this study, mothballed large portions of their deployed arsenals of these weapons systems.

A factor that exerted a clear and continuous influence upon the American (and thus Soviet) military decision makers, during the period of this study, was the Vietnam war. The Vietnam war, by changing the policy objectives of the United States, affected the deployment levels of U.S. weapons stocks. However, the decade of struggle in Vietnam corresponds roughly to the period of this study. Thus, the influence of the Vietnam conflict is felt, for the most part, continuously throughout the time series. Accounting for the variance provided by the Vietnam war would therefore not add much predictive power and would subtract from an already precariously low number of degrees of freedom. Therefore, I chose not to take the Vietnam conflict into account.
FIGURE 2
The Classic Arms Race: Soviet Armored Vehicles Against American Antitank Missiles
(Note Differing Scales)
FIGURE 3

The Classic Disarmaments Race 1: Soviet Interceptors Against American Bombers
(Note Differing Scales)
FIGURE 4
The Classic Disarmaments Race II: Soviet Strategic Bombers Against American Strategic Interceptors
(Note Differing Scales)
POLSICY COMPONENTS OF ARMS COMPETITION

The actual model to be estimated then for the testing of hypothesis \(H_1\) is of the following (in reduced form):

\[
\begin{align*}
w^1_{us} &= \alpha^1 + \beta^1_1 w^2_{su} + \beta^1_2 \text{GNP}_{su} + u^1, \\
\end{align*}
\]

\[
\begin{align*}
\alpha^2 + \beta^2_1 w^1_{us} + \beta^2_2 \text{GNP}_{us} + u^2, \\
\end{align*}
\]

where: \(w^1_{us}\) is the deployed level of weapons system 1 by the United States; \(w^2_{su}\) is the deployed level of weapons system 2 by the Soviet Union, in which the policy goals of weapons system 2 is incompatible with the policy goals of weapons system 1 of the United States; \(\alpha^1\) and \(\alpha^2\) are constant terms; GNP<sub>us</sub> and GNP<sub>su</sub> are the gross national product of the United States and Soviet Union, respectively; and \(u^1\) and \(u^2\) are error terms. The comparative statics of the model in the previous section, summarized in Equation (4), from which hypothesis \(H_1\) was deduced, offer clear predictions for the sign of the coefficients \(\beta^1_1\) and \(\beta^2_1\). The model predicts these coefficients to be positive.

The simultaneous equations system in Equation (5) is identified by exclusion restrictions on the exogenous (independent) variables. The coefficients were estimated by the method of two-stage least squares. In time series data, however, successive residuals tend to be highly correlated, resulting in biased estimates. The two-stage least squares method was therefore augmented by performing a first-order serial autoregressive process (quasi first-difference transformation) to the regression equations. Such a transformation implicitly assumes a first-order serial correlation of the residuals.\(^6\)

The results of this analysis are reported in Table 2. The top numbers in each entry are the unstandardized two-stage least square regression coefficients; the numbers in parenthesis below are the standard errors. Serial correlation may still pose a problem if present. The Durbin-Watson statistics reported in Table 2, though, are quite well behaved; in only one of the 12 equations are they less than 1.5 or greater than 2.5.

The estimated value of the correlation coefficient (\(\rho\)) employed in the first-difference transformation was reported for each equation as well. On the whole, these coefficients were small; in only one of the 12 equations did the estimated correlation between the untransformed residuals exceed .5 in absolute magnitude.

The figures in Table 2 provide strong support for hypothesis \(H_1\) and the decision theoretic model. Eight of the 12 interactive weapons coefficients were of the predicted sign (positive), and seven of these were significant at the .05 level. Only four were of the incorrect sign, and none were significantly different from zero. Moreover, the equations that

\(^6\) See Appendix for Equation (5) rewritten with residuals \(u^1\) and \(u^2\) autocorrelated and the resulting first-difference transformation.
<table>
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<tr>
<th>Dependent Endogenous Variable</th>
<th>Constant</th>
<th>Coefficient of Independent Endogenous Variable</th>
<th>Independent Endogenous Variable</th>
<th>Coefficient of Exogenous Variable GNP</th>
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<th>df</th>
<th>D-W$^a$</th>
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Note: The top numbers in each entry are the unstandardized two-stage least square regression coefficients; the numbers in parentheses below are the standard errors.

* D-W, Durbin-Watson statistic.

*Coefficient is significant at the 0.05 level.

**Coefficient is significant at the 0.01 level.
exhibited coefficients not in line with the predictions of the model were the equations with the smallest degrees of freedom (six to eight), and thus the results from these truncated time series may not reflect the true underlying relationship.

Interestingly, the estimation did rather poorly, with respect to hypothesis \( H_1 \), on the issue of the putative arms competition between American antitank missiles and Soviet heavy and medium tanks, as well as between American tactical aircraft and Soviet surface-to-air missiles. The lack of supportive evidence from the estimation of these equations is all the more intriguing in light of the appearance of an arms race between American antitank missiles and Soviet tanks in Figure 2. As stated above, however, these equations were estimated from a very small number of observations, and the resulting lack of degrees of freedom almost certainly curtailed the convergence of the estimated coefficients.

The production and technology constraint proxy, GNP, does not provide a consistent influence in estimating force levels. Though seven of 12 coefficients are significant, there is no consistent pattern to the signs of the estimated coefficients.

That arms races are evident between pairs of weapons systems deployed to achieve mutually incompatible policy goals is also supported by the specification of Equation (5) and the evidence in Table 2. Hypothesis \( H_2 \) suggests that we should not observe arms competitions between weapons systems whose policy goals are not mutually incompatible. In this regard, recall that Figure 1 had suggested that no arms competition was evident between American and Soviet manned strategic bombers. Indeed, these manned strategic bombers could, all else constant, carry out their policy objectives irrespective of the number of manned strategic bombers deployed by the other side.

Hypothesis \( H_2 \) was tested employing the same sample of weapons systems used to test hypothesis \( H_1 \). The model to be estimated to test \( H_2 \) is similar in form to the model estimated to test \( H_1 \):

\[
\begin{align*}
\omega_{1u} &= \alpha^1 + \beta^1_1 \omega^2_{su} + \beta^1_2 \text{GNP}_{su} + u^1, \\
\omega^1_{su} &= \alpha^2 + \beta^2_1 \omega^1_{us} + \beta^2_2 \text{GNP}_{us} + u^2,
\end{align*}
\]

where \( \omega^2_{us} \) is the deployed level of weapons system 1 by the United States; \( \omega^1_{su} \) is the deployed level of weapons system 1 by the Soviet Union of like variety to weapons system 1 of the United States; \( \alpha^1 \) and \( \alpha^2 \) are constant terms; GNP\(_{us}\) and GNP\(_{su}\) are gross national products of the United States and the Soviet Union, respectively; and \( u^1 \) and \( u^2 \) are error terms.

In this system, quantities of like varieties of weapons systems, between the United States and the USSR, were regressed on one another. It was assumed that the policy goals of similar weapons across nations
would not be incompatible. However, given the multiplicity of uses and policy objectives of these various weapons systems, such an assumption may indeed be quite strong. However any resulting bias would, in fact, be disfavored to the hypothesis under consideration, and so the analysis reported in Table 3 is indeed a strong test of hypothesis \( H_2 \).

Again, to summarize the expectations of hypothesis \( H_2 \), we expect the signs of the interactive weapons coefficients, \( \beta'_1 \) and \( \beta'_2 \), to be nonpositive. The simultaneous equations system in Equation (6) was estimated by the method of two-stage least squares, but adjusted by autoregression transformation to account for the presence of serial correlation. The results of the analysis are reported in Table 3.

The serial correlation problems posed by the estimation, though more severe than those reported in Table 2, are still quite mild. In only four of the 12 equations reported in Table 3 are the Durbin-Watson statistics less than 1.5 or greater than 2.5.

The coefficients reported in Table 3 strongly support the decision theoretic model and hypothesis \( H_2 \). The signs of the coefficients of the interactive weapons term in Table 3 unanimously support the prediction of the model. Though half (six of 12) of the coefficients are of the incorrect sign (positive), with respect to hypothesis \( H_2 \), none were statistically significant at the .05 level. The only two significant coefficients of the interactive weapons terms were, in fact, negative.

Though previous investigations have cast doubt on the general existence of Richardson arms races, the evidence presented here suggests that arms competitions, for two mutually antagonistic countries, between weapons systems with mutually incompatible policy goals do in fact exist. Further, the evidence presented suggests that arms competition will occur only between such weapons systems.

In general, the decision theoretic model of armament choice and hypotheses \( H_1 \) and \( H_2 \) are supported by the tests and model specification tendered here. Arms competitions, by this analysis, are a reflection of the foreign policy competition between the United States and the Soviet Union. The results of this estimation are all the more impressive given the small number of observations from which the estimates were derived.

**Conclusion**

A simple model of armament choice, in which military decision makers make trade-offs between alternative weapons deployments, was developed to provide a framework for relating the policy characteristics of weapons systems to their deployed levels. The resulting Nash bargaining game and the comparative statics of the optimization problem yielded a refutable hypothesis predicting the nature and structure of armament races: arms races will occur only between weapons systems with mutually incompatible policy missions.
TABLE 3
Test of Hypothesis $H_2$

<table>
<thead>
<tr>
<th>Dependent Endogenous Variable</th>
<th>Coefficient of Independent Endogenous Variable</th>
<th>Independent Endogenous Variable</th>
<th>Coefficient of Exogenous Variable</th>
<th>$\rho$</th>
<th>df</th>
<th>D–W*</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. strat bombers</td>
<td>681.6 (652.5)</td>
<td>0.40</td>
<td>SU strat bombers</td>
<td>−0.11</td>
<td>0.63</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>GNP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. strat intercept</td>
<td>828.6 (723.7)</td>
<td>0.18</td>
<td>SU strat intercept</td>
<td>−0.61</td>
<td>0.40</td>
<td>10</td>
</tr>
<tr>
<td>SU strat intercept</td>
<td>−1860.0 (5250.8)</td>
<td>4.16</td>
<td>U.S. strat intercept</td>
<td>1.86</td>
<td>0.42</td>
<td>10</td>
</tr>
<tr>
<td>U.S. tact aircraft</td>
<td>11388.4 (1816.2)**</td>
<td>−0.87</td>
<td>SU tact aircraft</td>
<td>−1.36</td>
<td>0.72</td>
<td>6</td>
</tr>
<tr>
<td>SU tact aircraft</td>
<td>2131.9 (5791.8)</td>
<td>−0.01</td>
<td>U.S. tact aircraft</td>
<td>1.99</td>
<td>0.21</td>
<td>6</td>
</tr>
<tr>
<td>U.S. tact S.A.M.</td>
<td>809.9 (762.1)</td>
<td>0.13</td>
<td>SU tact S.A.M.</td>
<td>−0.59</td>
<td>0.57</td>
<td>6</td>
</tr>
<tr>
<td>Dependent Endogenous Variable</td>
<td>Coefficient of Independent Endogenous Variable</td>
<td>Coefficient of Independent Endogenous Variable</td>
<td>Coefficient of Exogenous Variable GNP</td>
<td>ρ</td>
<td>df</td>
<td>D-W*</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------------------------</td>
<td>-----------------------------------------------</td>
<td>--------------------------------------</td>
<td>----</td>
<td>----</td>
<td>------</td>
</tr>
<tr>
<td>SU tact</td>
<td>-5228.0 (439.0)**</td>
<td>0.64 (1.60) U.S. tact</td>
<td>5.93 U.S. tact</td>
<td>-0.12</td>
<td>6</td>
<td>1.66</td>
</tr>
<tr>
<td>S.A.M.</td>
<td></td>
<td></td>
<td>S.A.M.</td>
<td>(0.37)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. H &amp; M tanks</td>
<td>5809.9 (1714.5)**</td>
<td>-2.06 (0.78)** SU H &amp; M tanks</td>
<td>2.10 SU H &amp; M tanks</td>
<td>-0.57</td>
<td>6</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S.A.M.</td>
<td>(0.95)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SU H &amp; M tanks</td>
<td>2775.2 (220.7)**</td>
<td>-0.45 (0.16)* U.S. H &amp; M tanks</td>
<td>1.03 U.S. H &amp; M tanks</td>
<td>-0.61</td>
<td>6</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S.A.M.</td>
<td>(0.08)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. anti-tank miss</td>
<td>-129139 (35769)**</td>
<td>0.24 (25.76) SU anti-tank miss</td>
<td>140.55 SU anti-tank miss</td>
<td>-0.06</td>
<td>6</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S.A.M.</td>
<td>(86.43)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SU anti-tank miss</td>
<td>1623.6 (1391.1)</td>
<td>0.00 (0.01) U.S. anti-tank miss</td>
<td>3.04 U.S. anti-tank miss</td>
<td>-0.57</td>
<td>6</td>
<td>1.91</td>
</tr>
</tbody>
</table>

Note: The top numbers in each entry are the unstandardized two-stage least square regression coefficients; the numbers in parentheses below are the standard errors.

* D-W, Durbin-Watson statistic.

**Coefficient is significant at the 0.01 level.
The simultaneous equations' regression analysis reported above supports the hypothesis that arms competitions exist only between weapons with incompatible policy missions. There did not appear to be any consistent influence from the exogenous variable GNP, although half (12) of the coefficients from Tables 2 and 3 were significant.

Lastly, the decision theoretic approach enables us to comment on the possibility of effective arms control. Previous arms control measures, such as the SALT I and SALT II agreements, have primarily established ceiling constraints upon weapons deployments. Such agreements (if enforceable) indeed limit the deployed levels of armaments. However, ceiling constraints act merely to alter the military decision makers' choice set over available weapons systems for which to achieve their defined policy objectives. In this framework, the decision makers will optimize around the constraint, according to their implicit rates of technical substitution between weapons (cf. McCubbins, 1982). The point here is that the addition of a ceiling constraint to the optimization problem defined in Equations (2) and (3), by eliminating a specific technology from the armament choice set, may lead to more dangerous arms competitions of higher technology as decision makers act to circumvent the arms constraint. It is doubtful, then, that effective arms control is achievable through ceiling constraints.

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APPENDIX

In equation (5) if the residuals $u^t$ and $u^2$ are autocorrelated, we can rewrite them as

$$u^{tr} = \rho u^{tr-1} + e^{tr}$$
$$u^{2t} = \rho u^{2t-1} + e^{2t},$$

where $u^{tr}$ is the residual of Equation (5) at time $t$; $\rho$ is the correlation between the residuals; $u^{tr-1}$ is the residual of Equation (5) at time $t - 1$; and $e^{tr}$ is the independent residual at time $t$. Similarly, for $u^{2t}$ the same is true.

A first-difference transformation of Equation (5) would then be

$$w^{tr}_u - \rho w^{tr-1}_u = \alpha^t (1 - \rho) + \beta^t (w^{t-1}_u - \rho w^{t-1}_u) + \beta^t (\text{GNP}^{t-1}_u - \rho \text{GNP}^{t-1}_u) + e^{tr}$$
$$w^{2t}_u - \rho w^{2t-1}_u = \alpha^t (1 - \rho) + \beta^t (w^{t-1}_u - \rho w^{t-1}_u) + \beta^t (\text{GNP}^{t-1}_u - \rho \text{GNP}^{t-1}_u) + e^{2t}.$$


